

Towards a model-independent partial wave analysis for pseudoscalar meson photoproduction

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Abstract. Amplitude and partial wave analyses for pion, eta or kaon photoproduction are discussed in the context of ‘complete experiments’. It is shown that the model-independent helicity amplitudes obtained from at least 8 polarization observables including beam, target and recoil polarization can not be used to determine underlying resonance parameters. However, a truncated partial wave analysis, which theoretically requires only 5 observables will be possible with minimal model input.

Keywords: complete experiment, partial wave analysis, pion photoproduction

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INTRODUCTION

Around the year 1970 people started to think about how to determine the four complex helicity amplitudes for pseudoscalar meson photoproduction from a complete set of experiments. In 1975 Barker, Donnachie and Storrow published their classical paper on ‘Complete Experiments’. After reconsiderations and careful studies of discrete ambiguities [1, 2, 3], in the 90s it became clear that such a model independent amplitude analysis would require at least 8 polarization observables which have to be carefully chosen. There are hundreds of possible combinations, but all of them would require a polarized beam and target and in addition also recoil polarization measurements. Technically this was not possible until very recently, when transverse polarized targets came into operation at Mainz, Bonn and JLab and furthermore recoil polarization measurements by nucleon rescattering has been shown to be doable.

COMPLETE EXPERIMENTS

A complete experiment is a set of measurements which is sufficient to predict all other possible experiments, provided that the measurements are free of uncertainties. Therefore it is first of all an academical problem, which can be solved by mathematical algorithms. In practise, however, it will not work in the same way and either a very high statistical precision would be required, which is very unlikely, or further measurements of other polarization observables are necessary. Both problems, first the mathematical problem but also the problem for a physical experiment can be studied with the help of state-of-the-art models like MAID or partial wave analyses (PWA) like SAID. With high precision calculations the complete sets of observables can be checked and with pseudo-data, generated from models and PWA, real experiments can be simulated under realistic conditions.

Amplitude analysis

Pseudoscalar meson photoproduction (as γ, π) has 8 spin degrees of freedom and due to parity conservation it can be described by 4 complex amplitudes of 2 kinematical variables. Possible sets of amplitudes are: Invariant amplitudes A_i , CGLN amplitudes F_i , helicity amplitudes H_i or transversity amplitudes b_i . All of them are linearly related to each other and further combinations are possible. Most often in the literature the helicity basis was chosen and the 16 possible polarization observables can be expressed in bilinear products

$$O_i(W, \theta) = \frac{q}{k} \sum_{k, \ell=1}^4 \alpha_{k, \ell} H_k(W, \theta) H_\ell^*(W, \theta), \quad (1)$$

where O_1 is the unpolarized differential cross section σ_0 and all other observables are products of asymmetries with σ_0 , for details see Table 1.

From a complete set of 8 measurements $\{O_i(W, \theta)\}$ one can only determine the moduli of the 4 amplitudes and 3 relative phases. However, there is always an unknown overall phase, e.g. $\phi_1(W, \theta)$, which can not be determined by additional measurements. Even with the help of unitarity in form of Watson's theorem this angle-dependent phase cannot be provided. This has very strong consequences, namely a partial wave decomposition would lead to wrong partial waves, which would be useless for nucleon resonance analysis.

Partial wave analysis

As the main goal in the data analysis of photoproduction is the search for nucleon resonances and their properties, one can directly perform a partial wave analysis from the observables without going through the underlying amplitudes. Such an analysis would be a truncated partial wave analysis with a minimal model dependence (i) from the truncation of the series at a maximal angular momentum ℓ_{max} and (ii) from an overall unknown phase as in the case of the amplitude analysis in the previous paragraph. However, in the PWA the overall phase would only be a function of energy and with additional theoretical help it can be constrained without strong model assumptions. Such a concept was already discussed and applied for γ, π in the 80s by Grushin [4] for a PWA in the region of the $\Delta(1232)$ resonance. In a 2-step process first γ, π^+ was analyzed in a truncation to S and P waves, where all higher partial waves were taken as Born terms, dominantly from the pion-pole contribution, which contributes only to charged pion production. This had the additional advantage that the overall phase was determined by the real Born amplitudes for $\ell > 1$. In order to separate the isospin 1/2 and 3/2 a further analysis has to be done for γ, π^0 . For neutral pion production, the higher partial waves are much smaller and can be neglected, however, in this case the overall phase remains undetermined. This problem could be solved by applying the Watson theorem just for

TABLE 1. Spin observables expressed by helicity amplitudes in the notation of Walker [5]. The sign definition is taken from Barker, Donnachie and Storrow [6] by replacing $N \rightarrow H_2$, $S_1 \rightarrow H_1$, $S_2 \rightarrow H_4$, $D \rightarrow H_3$. This sign definition is also used by SAID and MAID. The last column compares with Fasano, Tabakin and Saghai (FTS) [1], which is the second most often used sign definition in the literature. It has been adopted e.g. by recent work of Anisovich et al. [7] and Dey et al. [8], while in a paper by Sandorfi et al. [9], a definition very close to FTS is used, however, with \hat{E} changed into $-\hat{E}$. For the polarized observables the notation $\hat{\Sigma} := \sigma_0 \Sigma$ etc. is used and a factor q/k is dropped in all observables. Furthermore, the parameters $\alpha_i, \beta_i, \gamma_i$ for the $\cos\theta$ expansions in Eq. (3,5) are given.

| type | O_i | α_i | β_i | γ_i | helicity representation | Fasano et al. [1] |
|----------------|--------------------|------------|-----------|------------|-------------------------------------------------------|---------------------|
| \mathcal{S} | σ_0 | 0 | +1 | -2 | $\frac{1}{2}(H_1 ^2 + H_2 ^2 + H_3 ^2 + H_4 ^2)$ | $+\sigma_0$ |
| | $\hat{\Sigma}$ | 2 | -1 | -2 | $\text{Re}(H_1 H_4^* - H_2 H_3^*)$ | $+\hat{\Sigma}$ |
| | \hat{T} | 1 | 0 | -1 | $\text{Im}(H_1 H_2^* + H_3 H_4^*)$ | $+\hat{T}$ |
| | \hat{P} | 1 | 0 | -1 | $-\text{Im}(H_1 H_3^* + H_2 H_4^*)$ | $+\hat{P}$ |
| \mathcal{BT} | \hat{G} | 2 | -1 | -1 | $-\text{Im}(H_1 H_4^* + H_2 H_3^*)$ | $+\hat{G}$ |
| | \hat{H} | 1 | 0 | -1 | $-\text{Im}(H_1 H_3^* - H_2 H_4^*)$ | $-\hat{H}$ |
| | \hat{E} | 0 | +1 | -1 | $\frac{1}{2}(- H_1 ^2 + H_2 ^2 - H_3 ^2 + H_4 ^2)$ | $-\hat{E}$ |
| | \hat{F} | 1 | 0 | -1 | $\text{Re}(H_1 H_2^* + H_3 H_4^*)$ | $+\hat{F}$ |
| \mathcal{BR} | $\hat{O}_{\chi'}$ | 1 | +1 | -1 | $-\text{Im}(H_1 H_2^* - H_3 H_4^*)$ | $-\hat{O}_{\chi'}$ |
| | $\hat{O}_{\chi''}$ | 2 | 0 | -1 | $\text{Im}(H_1 H_4^* - H_2 H_3^*)$ | $-\hat{O}_{\chi''}$ |
| | $\hat{C}_{\chi'}$ | 1 | +1 | -1 | $-\text{Re}(H_1 H_3^* + H_2 H_4^*)$ | $-\hat{C}_{\chi'}$ |
| | $\hat{C}_{\chi''}$ | 0 | +2 | -1 | $\frac{1}{2}(- H_1 ^2 - H_2 ^2 + H_3 ^2 + H_4 ^2)$ | $-\hat{C}_{\chi''}$ |
| \mathcal{TR} | $\hat{T}_{\chi'}$ | 2 | 0 | -2 | $\text{Re}(H_1 H_4^* + H_2 H_3^*)$ | $+\hat{T}_{\chi'}$ |
| | $\hat{T}_{\chi''}$ | 1 | +1 | -2 | $\text{Re}(H_1 H_2^* - H_3 H_4^*)$ | $+\hat{T}_{\chi''}$ |
| | $\hat{L}_{\chi'}$ | 1 | +1 | -2 | $-\text{Re}(H_1 H_3^* - H_2 H_4^*)$ | $-\hat{L}_{\chi'}$ |
| | $\hat{L}_{\chi''}$ | 0 | +2 | -2 | $\frac{1}{2}(H_1 ^2 - H_2 ^2 - H_3 ^2 + H_4 ^2)$ | $+\hat{L}_{\chi''}$ |

the P_{33} partial wave in the following way [4, 10]

$$M_{1+}^{\pi^0 p} = \alpha e^{i\delta_{33}} + \frac{1}{\sqrt{2}} M_{1+}^{\pi^+ n}. \quad (2)$$

Formally, the truncated partial wave analysis can be performed in the following way. All observables can be expanded either in a Legendre series or in a $\cos\theta$ series, the first one has no real advantage, as we will see there is no physical angular momentum involved in the order of the power series

$$O_i(W, \theta) = \frac{q}{k} \sin^{\alpha_i} \theta \sum_{k=0}^{2\ell_{\max} + \beta_i} a_k^i(W) \cos^k \theta, \quad (3)$$

$$a_k^i(W) = \sum_{\ell, \ell'=0}^{\ell_{\max}} \sum_{k, k'=1}^4 \alpha_{\ell, \ell'}^{k, k'} \mathcal{M}_{\ell, k}(W) \mathcal{M}_{\ell', k'}^*(W), \quad (4)$$

where k, k' denote the 4 possible electric and magnetic multipoles for each πN angular momentum ℓ , namely $\mathcal{M}_{\ell, k} = \{E_{\ell+}, E_{\ell-}, M_{\ell+}, M_{\ell-}\}$. For an S, P wave truncation ($\ell_{\max} = 1$) there are 4 complex multipoles $E_{0+}, E_{1+}, M_{1+}, M_{1-}$ leading to 7 free real parameters and an arbitrary phase, which can be put to zero for the beginning. With the expansion

parameters α_i, β_i from Table 1 one can already get 8 observable coefficients from the first group of 4 \mathcal{S} -type observables. This already exceeds the number of free parameters, however, in order to resolve discrete ambiguities, one more observable or at least one more coefficient needs to be measured. This can be taken from any type of double polarization $\mathcal{BT}, \mathcal{BR}, \mathcal{TR}$. As has been shown by Omelaenko [11] the same is true for any PWA with truncation at ℓ_{max} . For the determination of the $8\ell_{max} - 1$ free parameters one has the possibility to measure $(8\ell_{max}, 8\ell_{max}, 8\ell_{max} + 4, 8\ell_{max} + 4)$ coefficients for types $(\mathcal{S}, \mathcal{BT}, \mathcal{BR}, \mathcal{TR})$, respectively.

However, this is only the mathematical issue of the problem. For any finite precision of data, the solution of the problem becomes more involved and instead of exact solutions of the sets of quadratic equations one has to search for a minimum similar to a chisquare fit. Then the global minimum, which may already be difficult to find does not necessarily give the correct solution. Therefore also local minima have to be considered and further techniques are needed to arrive at the correct solution. On the experimental side one has two possibilities to improve the situation, either to get the observables at higher precision or to measure further observables than mathematically required.

In Ref. [9] in an analysis of kaon photoproduction with real world data such a problem with multiple local minima has been faced and work is in progress. Similar problems will probably occur also in eta photoproduction, however, in pion photoproduction it will be different. There, the more difficult task to measure and analyze 2 charged channels, $p(\gamma, \pi^0)p$ and $p(\gamma, \pi^+)n$ also provides a chance to solve the problem with only a minimal model input. The big advantage in this reaction is the existence of the pion-pole term, which contributes only to charged pion channels and which gives large contributions for higher partial waves. Since the pion-nucleon coupling constant is well known, these higher partial waves can be very precisely calculated and furthermore their phases are known, namely zero for real Born terms. This fixes the overall phase for π^+n partial waves. In a subsequent analysis of π^0 photoproduction these higher Born terms do not contribute, but as mentioned before, the overall phase can then be connected to the π^+n phase, Eq. (2). Of course this connection needs to be done only for one phase, which naturally will be taken as the P_{33} phase, which is very well known and which is elastic even up to $W \approx 1550$ MeV, whereas other phases as P_{11} become already inelastic for $W \approx 1300$ MeV, close to the $\pi\pi$ threshold. This method was first applied in the analysis of Grushin [4].

In the presence of the t -channel pole contribution, the expansion of Eq. (3) must be modified by

$$O_i(W, \theta) = \frac{q}{k} \sin^{\alpha_i} \theta \sum_{k=\gamma_i}^{2\ell_{max}+\beta_i} a_k^i(W) \left(1 - \frac{q\pi^+}{\omega\pi^+} \cos\theta\right)^k, \quad (5)$$

which is an expansion around the pion pole. From the values of γ_i in Table 1 one can see that most observables start with a single-pole structure $\sim 1/\kappa$ with $\kappa = 1 - q/\omega \cos\theta$, whereas the unpolarized cross section σ_0 and the beam asymmetry $\hat{\Sigma}$ as well as all observables of \mathcal{BR} type include an additional double-pole term $\sim 1/\kappa^2$, which is, however, completely fixed by the Born terms, hence by the πN coupling constant. Therefore, only the coefficients $a_{-1}^i(W)$ have to be obtained from the angular distributions of the observables.

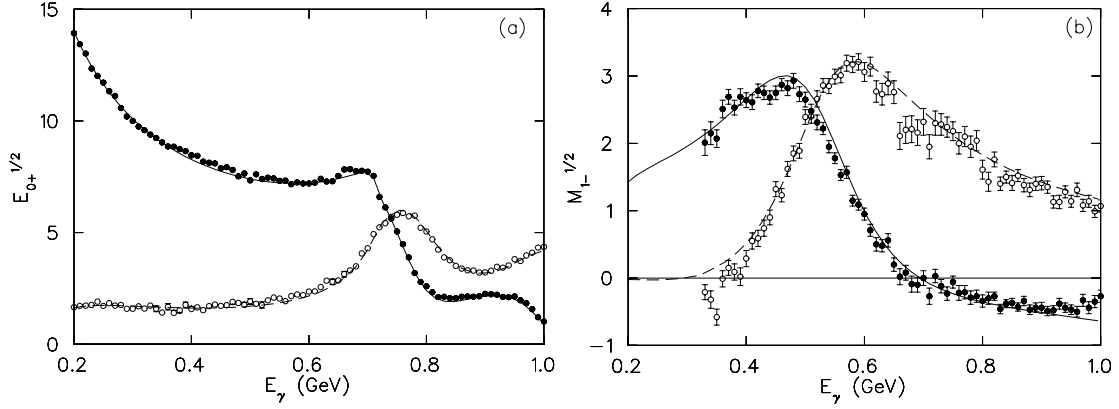


FIGURE 1. Real and imaginary parts of (a) the $S_{11}(pE)$ partial wave amplitude $E_{0+}^{1/2}$ and (b) the $P_{11}(pM)$ partial wave amplitude $M_{1-}^{1/2}$. The solid (dashed) line shows the real (imaginary) part of the MAID2007 solution, used for the pseudo-data generation. Solid (open) circles display real (imaginary) single-energy fits including the final phase searching (SE6p) to the following 6 observables without any recoil polarization measurement: $d\sigma/d\Omega$, two single-spin observables Σ and T and three beam-target double polarization observables E , F and G .

PARTIAL WAVE ANALYSIS WITH PSEUDO-DATA

In a first numerical attempt towards a model-independent partial wave analysis, a procedure similar to the second method described above has been applied [12], and pseudo-data, generated for γ, π^0 and γ, π^+ have been analyzed.

Events were generated over an energy range from $E_{lab} = 200 - 1200$ MeV and a full angular range of $\theta = 0 - 180^\circ$ for beam energy bins of $\Delta E_\gamma = 10$ MeV and angular bins of $\Delta\theta = 10^\circ$, based on the MAID2007 model predictions [13]. For each observable, typically $5 \cdot 10^6$ events have been generated over the full energy range. For each energy bin a single-energy (SE) analysis has been performed using the SAID PWA tools [14] in a 3-step process:

1. An energy-dependent analysis (ED) was obtained from starting values given by the current SAID SP09 ED solution.
2. By fixing the phases of the multipoles to the ED solution, only the absolute values of the S, P, D and F multipoles (ℓ_{max} depends on the energy) were searched.
3. In a final step also the phases of the multipoles were searched and a very satisfactory result was found in comparison with the underlying MAID2007 solution.

A series of fits have been performed [12] using 4, 6 and 8 observables. Here the example using 6 observables ($\sigma_0, \Sigma, T, E, F, G$) is demonstrated, where no recoil polarization has been used. As explained before, such an experiment would be incomplete in the sense of an ‘amplitude analysis’, but complete for a truncated partial wave analysis. In Fig. 1 two multipoles $E_{0+}^{1/2}$ and $M_{1-}^{1/2}$ for the S_{11} and P_{11} channels are shown and the SE6p fits of the final step in the analysis are compared to the MAID2007 solution. The fitted SE solutions are very close to the MAID ED solution with very small uncertainties for the S_{11} partial wave. For the P_{11} partial wave we obtain a larger statistical spread of

the SE solutions. This is typical for the $M_{1-}^{1/2}$ multipole, which is generally much more difficult to obtain with good accuracy [15, 13], because of the weaker sensitivity of the observables to this magnetic multipole. But also this multipole can be considerably improved in an analysis with 8 observables [12].

SUMMARY AND CONCLUSIONS

It is shown that for an analysis of N^* resonances, the amplitude analysis of a complete experiment is not very useful, because of an unknown energy and angle dependent phase that can not be determined by experiment and can not be provided by theory without a strong model dependence. However, the same measurements will be very useful for a truncated partial wave analysis with minimal model dependence due to truncations and extrapolations of Watson's theorem in the inelastic energy region. A further big advantage of such a PWA is a different counting of the necessary polarization observables, resulting in very different sets of observables. While it is certainly helpful to have polarization observables from 3 or 4 different types of Table 1, for a mathematical solution of the bilinear equations one can find minimal sets of only 5 observables from only 2 types, where either a polarized target or recoil polarization measurements can be completely avoided.

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